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COMPLEX MODEL OF THE EFFICIENCY OF RECTIFICATION PLATES. 7. INTERRELATIONSHIP OF INDIVIDUAL PARAMETERS

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Four regions of the complex mass-exchange model, the boundary cases of which are the Murphree model in analysis of the efficiencies in the vapor and liquid phases, the Hausen model, and the hypothetical model, are distinguished depending on the values of the distances h and h_1. The limits of variation of h and h_1 are found in each region. The dependences of h and h_1 on the activity coefficients of highly volatile and involatile components and the equilibrium constant are substantiated.

The relations for concurrent, countercurrent, and cross motion of the phases

$$\frac{L}{mV} = -\frac{2h_1 - 1 - \dfrac{1 - \dfrac{E_{\text{con}}}{E_{\text{con,m}}}}{1 - E_{\text{con}}}}{2h - 1 - \dfrac{1 - \dfrac{E_{\text{con}}}{E_{\text{con,m}}}}{1 - E_{\text{con}}}}\,, \tag{1}$$

$$\frac{L}{mV} = -\frac{2h_1 - 1 + \dfrac{1 - \dfrac{E_{\text{g}}}{E_{\text{g,m}}}}{1 - E_{\text{g}}}}{2h - 1 - \dfrac{1 - \dfrac{E_{\text{g}}}{E_{\text{g,m}}}}{1 - E_{\text{g}}}}\,, \tag{2}$$

$$\frac{L}{mV} = -\frac{2h_1 - 1}{2h - 1 - \dfrac{1 - \dfrac{E_{\text{k}}}{E_{\text{k,m}}}}{1 - E_{\text{k}}}}\,. \tag{3}$$

can be taken from [1–3].

Since under real conditions the ratio L/mV is positive, in formulas (1)–(3) at negative values of the numerators the denominators must be higher than zero and conversely. Moreover, theoretically this ratio can be equal to zero. The negative and zero numerators in (1)–(3) correspond to the conditions

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$$h_{1\mathrm{con}} \le \frac{1}{2} + \frac{1 - \dfrac{E_{\mathrm{con}}}{E_{\mathrm{con,m}}}}{2\,(1 - E_{\mathrm{con}})}\,, \tag{4}$$

$$h_{1\mathrm{g}} \le \frac{1}{2} - \frac{1 - \dfrac{E_{\mathrm{g}}}{E_{\mathrm{g,m}}}}{2\,(1 - E_{\mathrm{g}})}\,, \tag{5}$$

$$h_{1\mathrm{k}} \le \frac{1}{2}\,, \tag{6}$$

and the positive denominators correspond to

$$h \ge \frac{1}{2} + \frac{1 - \dfrac{E}{E_{\mathrm{m}}}}{2\,(1 - E)}\,. \tag{7}$$

The latter expression holds for concurrent, countercurrent, and cross motion.

Since the complex model must be used in all three forms of organization of flows, of formulas (4)–(6), condition (5) is the most general, i.e.,

$$h_1 \le \frac{1}{2} - \frac{1 - \dfrac{E}{E_{\mathrm{m}}}}{2\,(1 - E)}\,. \tag{8}$$

The positive numerators in (1)–(3) correspond to the conditions

$$h_{1\mathrm{con}} \ge \frac{1}{2} + \frac{1 - \dfrac{E_{\mathrm{con}}}{E_{\mathrm{con,m}}}}{2\,(1 - E_{\mathrm{con}})}\,, \quad h_{1\mathrm{k}} \ge \frac{1}{2}\,, \quad h_{1\mathrm{g}} \ge \frac{1}{2} - \frac{1 - \dfrac{E_{\mathrm{g}}}{E_{\mathrm{g,m}}}}{2\,(1 - E_{\mathrm{g}})}\,,$$

the most general of which has the form

$$h_1 \ge \frac{1}{2} + \frac{1 - \dfrac{E}{E_{\mathrm{m}}}}{2\,(1 - E)}\,, \tag{9}$$

and the negative denominators correspond to

$$h \le \frac{1}{2} + \frac{1 - \dfrac{E}{E_{\mathrm{m}}}}{2\,(1 - E)}\,. \tag{10}$$

Dependences (7)–(10) are obtained for $E < E_{\mathrm{m}}$. In the case of the inverse ratio of the efficiencies, from (1)–(3) at negative numerators and positive denominators we similarly obtain

$$h_1 \le \frac{1}{2} - \frac{\dfrac{E}{E_{\mathrm{m}}} - 1}{2\,(1 - E)}\,, \tag{11}$$

$$h \geq \frac{1}{2} - \frac{\dfrac{E}{E_\mathrm{m}} - 1}{2\,(1 - E)} \tag{12}$$

and at positive numerators and negative denominators we obtain

$$h_1 \geq \frac{1}{2} + \frac{\dfrac{E}{E_\mathrm{m}} - 1}{2\,(1 - E)}\,, \tag{13}$$

$$h \leq \frac{1}{2} - \frac{\dfrac{E}{E_\mathrm{m}} - 1}{2\,(1 - E)}\,. \tag{14}$$

Inequalities (7)–(14) refer to different regions of the complex model. Dependences (7) and (8) characterize that part of the model where the distances h_1 and h take on values within the ranges $0 \leq h_1 \leq 0.5$ and $1 \geq h \geq 0.5$. The boundary case of this region is the Murphree model [4–6] in analysis of the efficiency in the liquid, when $h_1 = 0$ and $h = 1$. Relations (9) and (10) differ in the ranges $1 \geq h_1 \geq 0.5$ and $0.5 \leq h \leq 1$. The limiting values are $h_1 = h = 1$, which refer to the hypothetical mass-exchange model presented earlier [7, 8]. Inequalities (11) and (12) refer to that part of the complex model where h_1 and h are minimum and in the limit are equal to zero, as is the case in the Hausen model [5, 6, 9]. Formulas (13) and (14) characterize the sector of the complex model the boundary of which is the Murphree model [4–6] in analysis of the efficiency in the vapor phase. Ordinary values of the distances in question are within the ranges $1 \geq h_1 \geq 0.5$ and $0 \leq h \leq 0.5$.

Another limit of the mentioned regions of the complex model is given by the values $h_1 = h = 0.5$ which take place in separation of ideal mixtures.

It is shown in [1–3] that the efficiency of mass exchange decreases with increase in the distances h_1 and h. Inequalities (8) and (13) drop out of this regularity; in the first of these inequalities, low values of h_1 correspond to low efficiencies, while in the second inequality high values of h_1 correspond to the high values of E. These inequalities reflect the regions of the complex model where, in both variants, the boundary cases are the Murphree model. The probability of existence of these regions is caused by the fact that in many computational relations one uses the combination $(hL/mV + h_1)$, where a decrease in one distance is compensated for with a decrease in the other, which, however, introduces certain errors into the results obtained. Probably, due to the mentioned peculiarities some researchers treat the Murphree model critically and prefer other models.

From (1)–(3) we can obtain the dependences of the distances h_1 and h on the main parameters of the complex model in concurrent, countercurrent, and cross motion of a vapor and a liquid:

$$h\frac{L}{mV} + h_1 = \frac{1}{2}\left(\frac{L}{mV} + 1\right) + \frac{1}{2}\left(\frac{L}{mV} + 1\right)\frac{1 - \dfrac{E_\mathrm{con}}{E_\mathrm{con,m}}}{1 - E_\mathrm{con}}\,, \tag{15}$$

$$h\frac{L}{mV} + h_1 = \frac{1}{2}\left(\frac{L}{mV} + 1\right) + \frac{1}{2}\left(\frac{L}{mV} - 1\right)\frac{1 - \dfrac{E_\mathrm{g}}{E_\mathrm{g,m}}}{1 - E_\mathrm{g}}\,, \tag{16}$$

$$h\frac{L}{mV}+h_1=\frac{1}{2}\left(\frac{L}{mV}+1\right)+\frac{L}{2mV}\frac{1-\dfrac{E_{\mathrm{k}}}{E_{\mathrm{k,m}}}}{1-E_{\mathrm{k}}}\,. \qquad (17)$$

On substitution of the limiting value $h = 1$ into (15), the other distance will be determined by the dependence

$$h_1=\frac{E_{\mathrm{con,m}}\,(1-E_{\mathrm{con}})+E_{\mathrm{con,m}}-E_{\mathrm{con}}}{2E_{\mathrm{con,m}}\,(1-E_{\mathrm{con}})}-\frac{L}{mV}\frac{E_{\mathrm{con}}\,(1-E_{\mathrm{con,m}})}{2E_{\mathrm{con,m}}\,(1-E_{\mathrm{con}})}\,. \qquad (18)$$

Assuming that $L/mV = 1$, we can simplify the obtained expression:

$$h_1=\frac{E_{\mathrm{con,m}}-E_{\mathrm{con}}}{E_{\mathrm{con,m}}\,(1-E_{\mathrm{con}})}\,. \qquad (19)$$

It is seen from (19) that the positive values of h_1 are attainable when $E_{\mathrm{con,m}} > E_{\mathrm{con}}$ and are impossible when $E_{\mathrm{con}} > E_{\mathrm{con,m}}$. For the real values of h_1 to be obtained, it is necessary to decrease L/mV in (18) or simultaneously h and L/mV in (15).

If in formula (15) we set $h_1 = 1$, as a result we have

$$h\frac{L}{mV}=\frac{L}{mV}\frac{E_{\mathrm{con,m}}\,(1-E_{\mathrm{con}})+E_{\mathrm{con,m}}-E_{\mathrm{con}}}{2E_{\mathrm{con,m}}\,(1-E_{\mathrm{con}})}-\frac{E_{\mathrm{con}}\,(1-E_{\mathrm{con,m}})}{2E_{\mathrm{con,m}}\,(1-E_{\mathrm{con}})}\,. \qquad (20)$$

Substitution of $L/mV = 1$ simplifies dependence (20) to a form similar to (19):

$$h=\frac{E_{\mathrm{con,m}}-E_{\mathrm{con}}}{E_{\mathrm{con,m}}\,(1-E_{\mathrm{con}})}\,. \qquad (21)$$

The positive values of h in formula (21) are also impossible when $E_{\mathrm{con}} > E_{\mathrm{con,m}}$. To eliminate this situation, it is necessary, in contrast to (18), to increase L/mV in (20). The contradiction mentioned indicates that the real values of one considered distance are attainable if the other is less than unity, irrespective of the values of L/mV.

Setting alternately the values $h = 1$ and $h_1 = 1$ in (16), we, correspondingly, obtain

$$h_1=\left(\frac{L}{mV}-1\right)\frac{E_{\mathrm{g}}\,(E_{\mathrm{g,m}}-1)}{2E_{\mathrm{g,m}}\,(1-E_{\mathrm{g}})}\,, \qquad (22)$$

$$h=\left(1-\frac{mV}{L}\right)\frac{E_{\mathrm{g,m}}(1-E_{\mathrm{g}})+E_{\mathrm{g,m}}-E_{\mathrm{g}}}{2E_{\mathrm{g,m}}\,(1-E_{\mathrm{g}})}\,. \qquad (23)$$

The first of the dependences derived shows the impossibility of the existence of positive values of h_1 irrespective of the ratio L/mV and the ratio of the efficiencies $E_{\mathrm{g,m}}$ and E_{g}. Consequently, the real values of h_1 are attainable at values of h knowingly less than unity. An analysis of expression (23) shows that the real values of h depend on the ratio of the efficiencies and are impossible when $E_{\mathrm{g,m}} < 1$. Therefore, the values of h_1 must also be less than unity. The indicated peculiarities are confirmed by Figs. 5 and 6 from [2], which do not have real values of the efficiencies at $h = 0.5$ and $h_1 > 0.75$, at $h_1 = 0.5$ and $h > 0.666$, and at $h = h_1 > 0.6$ for $L/mV = 1.5$

Substitution of h, h_1, and L/mV that are equal to unity into (17) leads to formulas similar to (19) and (21) with the only difference being that in cross motion of the vapor and the liquid the distances h_1 and h are half as large as in concurrent motion. Consequently, these distances can be positive under the conditions specified for the concurrent flow.

Since subtraction of the quantity L/mV, which in countercurrent flow must be more than unity, from the right-hand side of (16) leads to nonreal h_1 and subtraction of a value which is slightly less than unity leads to acceptable values of h, it is reasonable to decrease the quantity h by L/mV. This would broaden the possibilities of the complex model with respect to the distance h but simultaneously bound h_1 the limiting value of which under this assumption cannot exceed mV/L. However, the mentioned peculiarities of the interrelationship between h and h_1 are typical of only the countercurrent flow and are insignificant for the concurrent and cross motion of the phases. Moreover, it is seen from Fig. 5a and b [1, 2] and Fig. 4a and b [3] that changes in h and h_1 exert virtually the same effect on the ratio of the efficiencies E and E_m. The slight difference in the indicated pairs of graphs is caused by the factor L/mV at h, e.g., on the right-hand sides of dependences (15)–(17). And finally, as follows from [10], at the same $L/mV = 1.5$ mixing of the liquid extends the range of possible changes in the quantities h and h_1 which at $\varphi = 0.5$ varies from 0 to 0.8 compared to their maximum values of 0.6 in the absence of mixing. Countercurrent motion without mixing virtually does not occur. Therefore, for the regions of the complex model the limiting cases of which are the Hausen model and the hypothetical model, we can take the equality of the distances h and h_1.

Except for those mentioned, the values of h and h_1 for the regions whose limiting cases are the Hausen model and the hypothetical model must correspond to the following requirements:

(a) with increase in the activity of a highly volatile component, h must tend to zero, whereas with decrease in it, it must tend to unity;

(b) with increase in the activity of an involatile component, h_1 must tend to unity, whereas with decrease in it, it must tend to zero;

(c) in separation of mixtures approaching ideal ones, when the activity coefficients of both components approach unity, h and h_1 must be close to 0.5.

The conditions stated allow one to formulate the following dependence for determination of the distances h and h_1:

$$h = h_1 = \frac{\gamma_{\mathrm{in}}}{\gamma_{\mathrm{h}} + \gamma_{\mathrm{in}}} = \frac{1}{\dfrac{\gamma_{\mathrm{h}}}{\gamma_{\mathrm{in}}} + 1}. \tag{24}$$

Instead of the activity coefficients, in (24) we can also use the partial pressures of the highly volatile and involatile components. However, since both quantities are a rarity in the literature and data on the coefficients of equilibrium are more readily available and have received wide recognition from practical applications, for determination of the distances h and h_1 it is acceptable to use the expression

$$h = h_1 = \frac{1}{m + 1}. \tag{25}$$

Thus, with increase in the activity coefficient of the highly volatile component, decrease in the corresponding quantity of the involatile component, and increase in the equilibrium constant the distances h and h_1 decrease and the complex model shifts toward the Hausen model [5, 6, 9]. As γ_{in} increases, γ_{h} and m decrease and the complex model tends to the hypothetical model. If the activities of both components increase or decrease simultaneously, then a comparative analysis of their activities is necessary. Depending on the predominant effect of one of these components, the mass-exchange model can deviate toward either the

Hausen model or the hypothetical model. Thus, depending on the activity of the components of the separated mixture or the coefficient of equilibrium, we determine the mass-exchange model, which is individual for a specific mixture but can also change from plate to plate for a given solution as the activity of its components diverges from the initial values.

In the Murphree model, as the activity of the highly volatile component increases and that of the involatile component decreases, h_1 must tend to unity, and with the opposite tendency in the activities of the components it must tend to zero, which leads to the dependence

$$h_1 = \frac{\gamma_{\rm h}}{\gamma_{\rm h} + \gamma_{\rm in}} = \frac{1}{\dfrac{\gamma_{\rm in}}{\gamma_{\rm h}} + 1} \tag{26}$$

or

$$h_1 = \frac{m}{m + 1}\,. \tag{27}$$

In analysis of the efficiency in the vapor and liquid phases, the distance h for this model is determined from formulas (24) and (25).

NOTATION

E, efficiency of the plate; $\gamma_{\rm h}$ and $\gamma_{\rm in}$, activity coefficients of the highly volatile and involatile components of a real solution, respectively; φ, degree of mixing; h and h_1, dimensionless distance from the site of supply of the vapor and the liquid, respectively, to the surface where the concentrations of the phases on ideal and real plates are equal; L, molar flow of the liquid; m, equilibrium constant; V, molar flow of the vapor. Subscripts: g, counterflow; k, cross motion; m, values of the parameters at $h = h_1 = 0.5$; con, concurrent flow.

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